Analysis of the Methods used in "Perception of Climate Change"
[Hansen et al., 2012]

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Abstract

In a recent paper, Hansen et al. [2012] used new mathematical methods to study climate change, and showed that extreme temperatures are becoming more common. Moreover, histograms of temperature distributions show a widening with time. This result is dramatic, but easily misinterpreted to imply that the variability of temperature is increasing with time. In fact, even if the variance does not increase with time, the distributions used by Hansen et al. often show a broadening. We trace this effect to Hansen et al.'s approach of using a *base period* to define climate, in which all records are set to have the same average during a particular time period. This approach results in a phenomenon we call the 'pinch' effect, which tends to artificially reduce the variance during the base period, and gives the illusion that the variance is larger at other times. If the base period is chosen early in the record, then the variance appears to *increase* with time. However, if the base period is chosen at the end of the record, then the same analysis appears to show a variance that is *decreasing* with time. The results of Hansen et al. are correct: due to global warming we are now experiencing warm periods that are greater than one would expect without warming; however, this conclusion should not be interpreted to mean that the variability of temperature is increasing.

The Approach of Hansen et al.

In their recent paper, Hansen et al. [2012] study the global temperature change in the last 61 years by comparing it with the historic variability of temperatures. The main method is the following. They define their base period as 1951-1980. Temperatures at other times are all taken relative to this period, that is, the average temperature from 1951 – 1980 is subtracted. The difference is referred to as the temperature anomaly. To normalize these anomalies, Hansen et al. divide them by sigma, the standard deviation of the anomalies in the base period.

The main finding of the paper by Hansen et al. [2012] is that these distributions of T/sigma values shift to larger values and get broader for the later decades of the last 60 years (Fig. 1). From this Hansen et al. [2012] conclude "...that extreme anomalies such as those in Texas and Oklahoma in 2011 and Moscow in 2010 were a consequence of global warming because their likelihood in the absence of global warming was exceedingly small", referring to the fact that greater than 3 sigma events generally have a very small probability of occurring. They also find that the broadening of the distribution depends on the base period during which local temperatures are set to zero.

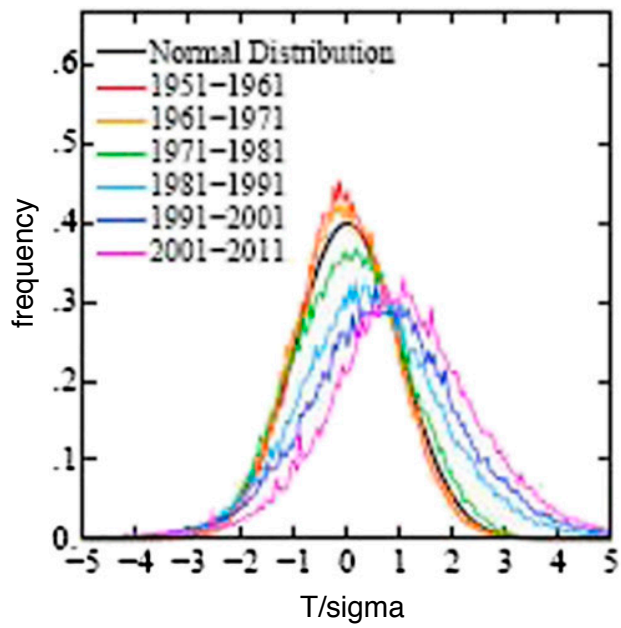


Figure 1: *From figure 4 of Hansen et al. [2012], a key result of the paper. The distributions are generated from a collection of local mean June-July-August temperature anomalies. At each location the temperature during the base period 1951-1980 is set to zero, then every such newly defined local temperature anomaly of the years 1951-2010 is divided by the local temporal standard deviation of the temperature anomalies during the base period, here 1951-1980. Then values for 11 year periods are collected and plotted as normalized distributions. Note that for later decades the mean of the distribution shifts to larger values and that the distributions are getting wider.*

It is clear in Figure 1 why this is so. As an example, let's take T/sigma = 3. As can be seen in the figure, the probability of such an event for the period 1951-1961 is extremely small; this is not surprising, since it corresponds to three standard deviations. For a normal distribution, the probability of a three sigma or greater event is 0.3%. Yet for the recent data 2001-2011, the frequency is quite high, about 0.1 in amplitude on this plot.

This rising probability might be due to one of several effects. If the temperature rises, but doesn't broaden, then events that were once rare will become common. It is also true that if the temperature doesn't rise, but simply broadens, then the rare events also can become common. Looking at figure one, it is tempting to think that both effects are occurring: the mean of the distribution is increasing and the distribution is broadening. However, this is an over-interpretation of the curves. In fact, even if the temperature variance were holding steady, the distribution of the variable T/sigma will show a broadening. In this memo we will show how this comes about. Thus we must be very cautious in interpreting the distribution.

We begin by using simulated data to show how three different effects: a range of different sigma, a range in different trends, or an increase in temporal

variability can lead to the broadening similar to that seen in Fig 1. These effects also lead to a base period dependence, such as that reported by Hansen et al. We then apply these insights when we turn to the temperature data used by Hansen et al. to show which of these mechanisms are responsible for the behavior seen in Figs 1 and 2.

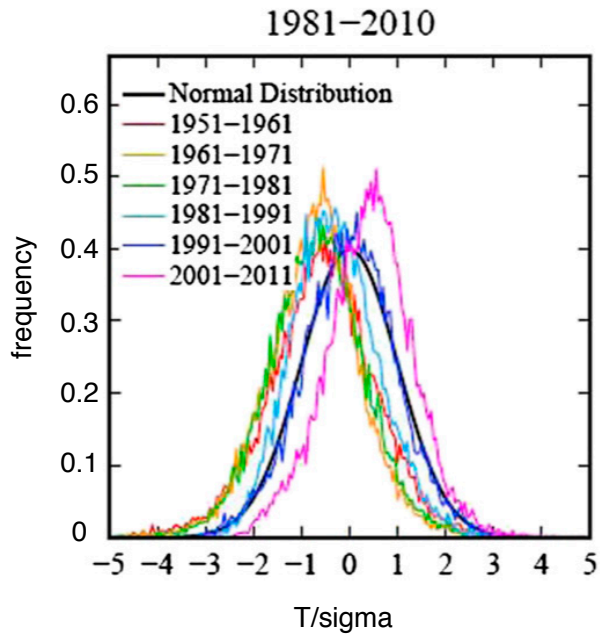


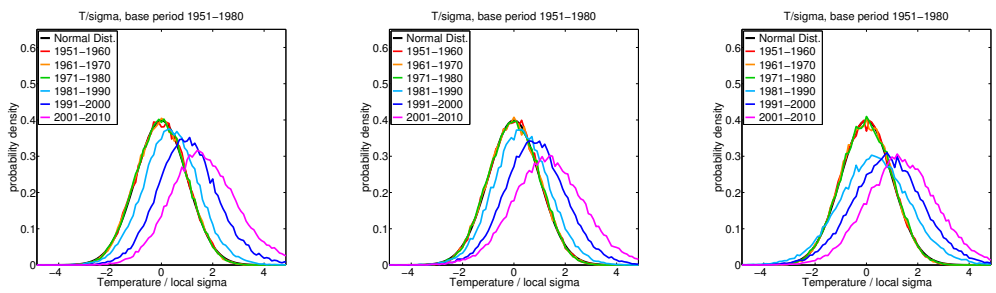
Figure 2: *From figure 9 of Hansen et al. [2012]. The T/sigma distributions are similar to the ones in Fig 1 but here the base period is 1981-2010, i.e. the temperature anomalies for each location are set to zero for that period instead of for the period of 1951-1980 (used in Fig 1). This figure also uses the sigma of 1981-2010 (although that is not the important factor for its appearance). Note that unlike those of Figure 1, these distributions do not widen for later time intervals.*

Understanding Broadening in the T/sigma Distribution

Once a base period is chosen, and the temperature anomaly average is set to zero for each temperature record during that period, one of the main causes of variation between sites will be removed. The temperature records during the base period are all forced to have the same mean, but only during the base period. That means that the variance over a longer period will normally be larger; the variance during the base period will appear "pinched" compared to other periods. This creates a narrow distribution of temperature anomalies within the base period and wider distributions outside. This is true for any system with spatially noisy data, e.g. where different locations have different temperature trends or different sigma. In such systems, forcing the average temperature during the base period to be zero for all locations will always lead to less spatial variation during the base period while increasing the spatial variation outside of the base period.

In a separate Berkeley Earth memo, Zeke Hausfather shows by analysis of the Berkeley Earth data compilation that temporal variability of temperature is not increasing, and can therefore not be the cause of the widening of the temperature distributions observed by Hansen et al. Here will use simulated data to show how the pinch effect can account for the broadening of the Hansen et al. distributions.

Before we get into the detailed analysis, let's look at some key results to show where we are headed. In Figure 3 we have used simulated data. In only one of these does the temperature variation increase, yet all show similar broadening; all three look similar to Figure 1 in Hansen et al. For the other two plots, the broadening of the distribution over time can be thought of as an artifact of the choice of base period; if chosen at the beginning of the records, then the pinch effect occurs there, and the distribution will broaden over time even though the variance in the temperature record does not.



(a) *different sigma* (b) *different local trends* (c) *temporal variability*

Figure 3: *For simulated data, three different mechanisms can produce similar widening of the T/sigma distribution as in Fig. 1. Here (a) is produced with a range of different sigma, (b) is produced with a range of different local trends and (c) is produced with increasing local temporal variability. The widening for the last one is abrupt because we chose a simple stepwise increase of temporal variability.*

Analysis Using Simulated Data

In this section we study the effect of different widening mechanisms on the T/sigma distributions as in Fig. 3. In order to apply different mechanisms to the T/sigma distribution in a controlled fashion we simulate a number of location dependent yearly mean June-July-August temperature anomalies for the years 1951-2010. To simulate different local temperature series we start with a temperature anomaly 'seed' function and add normal distributed random fluctuations. Here we apply the 'seed' attribute to each input parameter that is needed to produce random data.

The global temperature anomaly seed function is always taken to be zero in the period 1951-1980 and then either changes linearly or as a step function (Fig. 4). We will cover both of these cases separately. The step function provides a simple

model that we can use to study the effects of the broadening mechanisms without complications from a linear trend. However, using the linear trend provides a better way to compare the simulated data to real data. The seed functions are good approximations to the Berkeley Earth global average (Fig. 4).

In this memo, even when changing the base period, we will always use a fixed value for sigma. (We take it to be the sigma of the period 1951-1980.) We do this for simplicity, because changing it doesn't have a large effect on the T/sigma distribution, other than on its scale. This has been demonstrated in figure 4 of Hansen et al. [2012]. We will also work with base periods as Hansen et al. do by setting the local temperature anomalies to zero within the particular base period. For convenience, we will use 10-year periods instead of 11-year periods, but this doesn't impact the results. Whenever we generate random numbers based on input parameters we will label the input parameters, like sigma and trend, as 'seed' parameters. We will mostly discuss the temperature anomalies defined relative to some local base period, and will sometimes refer to this anomaly as T or temperature. We will emphasize when we mean actual temperatures instead of anomalies.

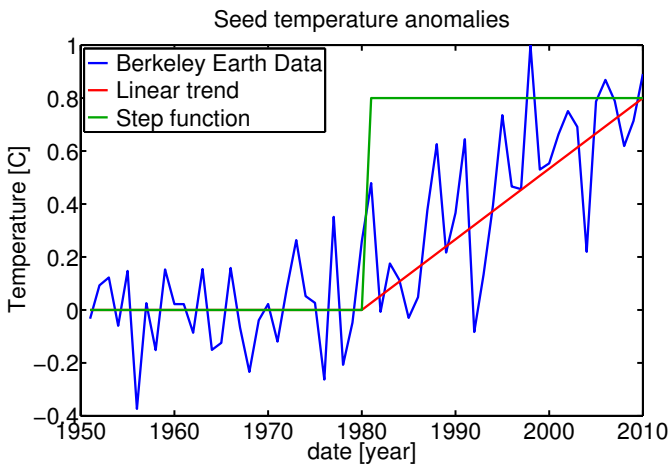


Figure 4: *We use two different seed temperature functions, a step function which avoids the complication of a linear trend while providing valuable insight into the various widening mechanisms of the T/sigma distributions and a linear trend for 1981-2010, which is more realistic as it is a reasonable approximation to the Berkeley Earth global temperature anomalies.*

To study the three mechanisms that cause widening we add random fluctuations to the global anomaly seed function in three different ways:

1. A range of different local seed anomaly trends or steps for 1981-2010. The seed sigma is the same for all locations and constant in time.

2. A range of different seed sigma for the 1951-2010 period, keeping each individual seed sigma fixed over the whole time period. The anomaly seed function is the same for all locations.

3. A seed sigma that is increasing in time. Seed anomaly function and seed sigma are the same for all locations. We will refer to this mechanism as increasing temporal variability.

In the following, we will first use a step function as the seed anomaly, in order to gain appreciation for the effect of higher temperature anomalies at later times without taking into account the added complexity of a linear trend. Then we will expand our analysis to a seed anomaly function with linear trend (Fig. 4).

Global Temperature Anomaly - Step Function

In this section we demonstrate the three widening mechanisms in the simple case of a step seed anomaly function (Fig. 4). To do this we simulate yearly data by adding random normal-distributed noise to this seed function. We will first discuss the widening mechanism of a range of different local trends, then a range of different sigma and finally an increasing temporal variability.

Different Local Seed Anomaly Trends

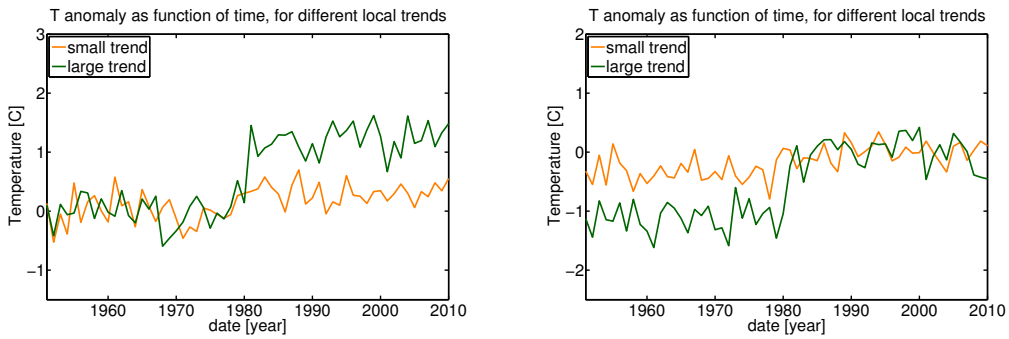
Two different local trends: As a case study for how different trends can cause a widening of the T/sigma distribution as well as a base period dependence, we will start with just two locations with different trends, a large step and a small step at 1981.

If the base period is 1951-1980, both locations will have zero mean in that period and then different anomalies at later times (Fig 5a). If however, the base period is 1981-2010, both locations will have zero mean during that period and differ for earlier times (Fig. 5b).

When choosing the 1951-1980 base period we can see what effect this will have on the combined T distribution of these two locations by simulating many anomaly series for each of the two seed sigma and then plotting the distributions of the anomalies based on each seed sigma separately (Fig. 6). This will give us a sense of the probability density function from which the 10 yearly anomaly values for a location with a particular seed sigma are picked. When using an early base period these distributions will be centered at different temperature anomalies for different locations. Therefore, when the two locations are put into the same distributions, when essentially the two probability density functions are added up, we will arrive at a wider distribution than for the earlier anomalies. The same argument leads to a wider early anomaly distribution if we choose a late base period. For this section we are assuming the same seed sigma for all locations, so dividing the anomaly by sigma would only scale the values, but not qualitatively change the effect.

Continuous distribution of local trends: Now that we know how two different trends can in principle lead to a widened distribution we can visualize this by using a continuous range of normal distributed trends. We now allow a number of different trends and arrive at narrow T distributions for times near or in the base period and widened distribution for times away from the base period (Fig. 7). This is due to the 'pinch' effect of forcing all anomalies within a certain period on the same local mean.

We will come back to these signatures of Fig 7 to analyze the real temperature data. A presence of this effect in the real data indicates a range of different local trends as the cause of the widening.



(a) *base period 1951-1980* **(b)** *base period 1981-2010*

Figure 5: *The anomalies of two different locations, one with a small trend and one with a large trend, are similar during the base period, because we force their mean during that time to be the same, i.e. zero. For times outside of the base period the anomalies are no longer constrained and can therefore be different from each other.*

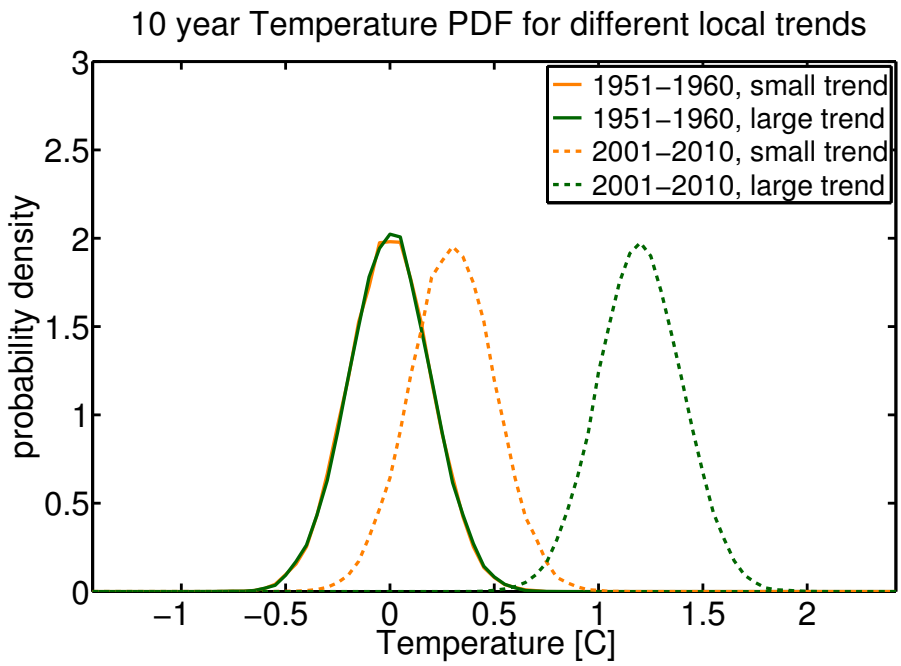
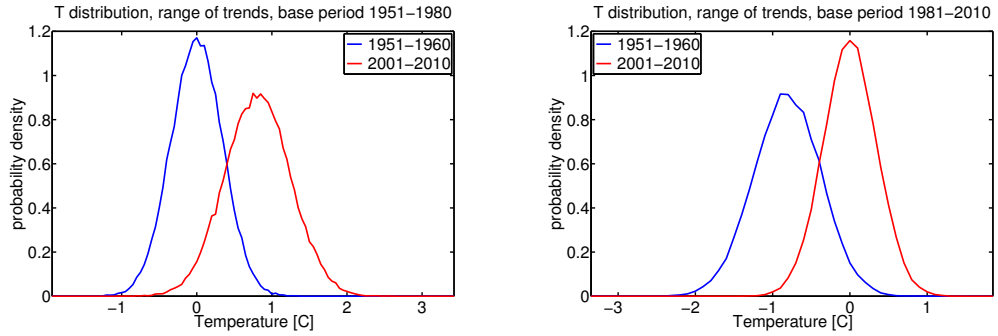


Figure 6: *By simulating many temperature series with one of the two trends and collecting them in a distribution we arrive at probability density functions from which an anomaly value for a location with a particular trend is drawn. The base period is 1951-1980, and the distributions for a period within the base period (solid) are the same, since we forced the two locations to have the same mean during that time. If the seed anomaly function wasn't perfectly flat during 1951-1980 and we allowed for different trends during that period there would be a slight difference between the two solid curves. However, in general the difference is larger the more time has passed before or after the base period, because the different local trends then have more time to separate after being 'pinched' together in the base period. We see this effect for the dashed curves, which are the anomaly distributions for two different trends. They differ substantially and if added up they yield a broader distribution than the sum of the solid curves. If the base period is 1981-2010, we would see the two solid curves separate and the dashed curves align.*



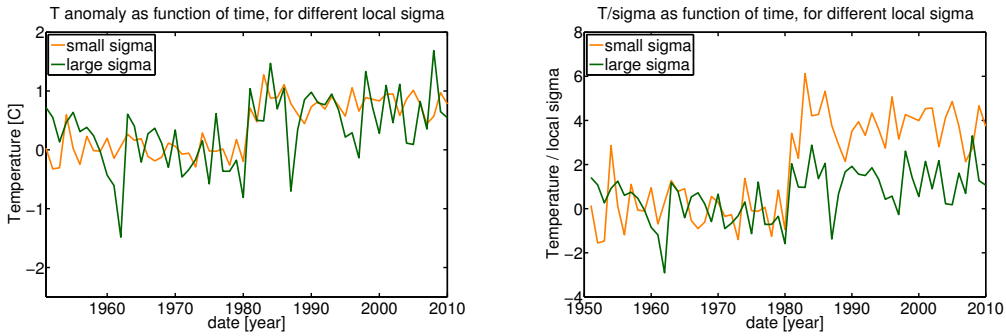
(a) (b)

Figure 7: *Simulated data with a range of different trends for the 1981-2010 period results in wider T distributions for times outside the base period. (a) early base period, so the later times have wider distributions, (b) late base period, so the earlier times have a wider distribution. This is due to the 'pinch' effect, which forces anomalies from different locations to the same mean in the base period thereby narrowing the T distribution within the base period relative to times outside the base period.*

Different Seed Sigma

Two different sigma: Let us again begin with a case study of two different sigmas, a large and a small one. No matter what the base period is, the temperature anomalies will be the same on average for both locations during every time period (Fig. 8a). However, if we scale the anomalies by dividing them by sigma, the resulting T/sigma series for the two locations are no longer the same in every time period (Fig. 8b). Now T/sigma for the location with smaller sigma increases more if the base period is 1951-1980. In contrast, if the base period is 1981-2010 the early times for the location with small sigma will have larger negative T/sigma values.

This will affect the global T/sigma distribution in a way that is made obvious when calculating the probability density function from which a local T/sigma series at a later decade will be drawn, by simulating a large number of data with two different seed sigma (Fig. 9). The two PDFs for small and large seed sigma in 1951-1980 have roughly the same width, but are shifted relative to each other. This means, that when T/sigma values from these two different locations are added up, the resulting global T/sigma distribution will be wider in later years, when the anomalies have increased. For a range of different seed sigma, as is more realistic for real data, the same effect will apply and the T/sigma distribution will get wider, simply because the values in the distribution are drawn from local probability density functions that are shifted and will add up to produce a widened distribution.



(a) (b)

Figure 8: *Simulated data for two locations with the same seed anomaly function but different seed sigma, the base period is 1951-1980. (a) The temperature anomaly series for the two locations have the same average increase in T, but their sigma are different. The T distribution for early and late times should therefore have different widths. (b) After dividing the temperature anomalies at every point in time by the appropriate sigma, the location with smaller sigma now has a larger increase of T/sigma and we therefore expect a broad range of different T/sigma values and a widening of the distribution at later times. For a base period of 1981-2010, the early times series of T/sigma would be different for the two locations. We again observe the 'pinch' effect, but now only for T/sigma, not for T.*

Continuous distribution of sigmas: The real temperature data have a range of different sigma (Fig 10), so we next use a continuous distribution of different seed sigma (log-normally distributed) to produce the random normal distributed noise on top of a common seed anomaly function. As before, we take these seed sigma to be constant in time.

We take a large number of local anomaly trends with random fluctuations based on our distribution of seed sigma and arrive at a widened T/sigma distribution for the later decade (Fig 11a). An important ingredient for the widening is that the temperature anomaly is different from zero, so that the different locations can register different shifts for their local T/sigma distribution to cause the broadening of the overall T/sigma distribution. Therefore, if we redefine the local mean of 1981-2010 to be zero, the earlier decade will have T values that are different from zero and therefore exhibit a widened global T/sigma distribution (Fig 11b). This is the 'pinch' effect for T/sigma and a range of sigma.

We will come back to these signatures of Fig 11 to analyze the real temperature data, because their presence in the real data would indicate a range of sigmas as the cause of the widening.

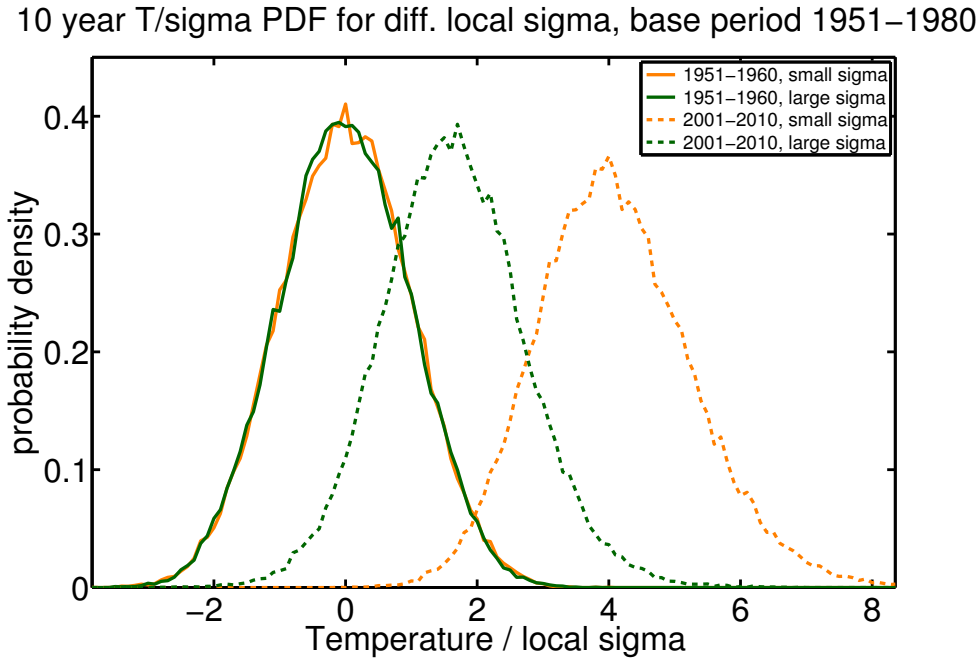


Figure 9: *The probability density functions from which local T/sigma values are drawn. The anomaly trend is the same for these two locations, one with small sigma, one with large sigma, so we wouldn't expect any difference between the early (solid) and late (dashed) distributions for T only. However, when the temperature anomaly is scaled by sigma, the smaller sigma location will have larger values of T/sigma at later times. The result is that the two distributions for two locations separate, which will result in a wider total T/sigma distribution when many locations' distributions are added up.*

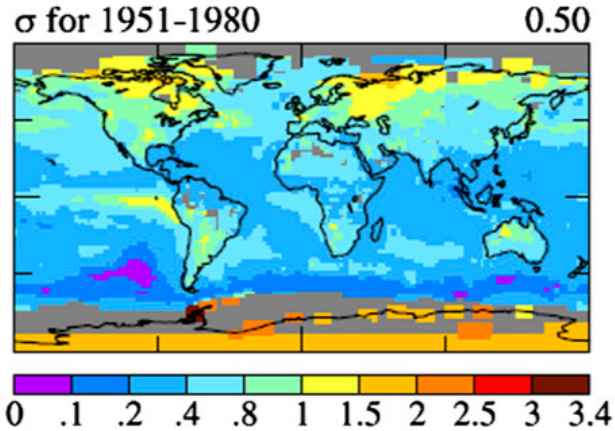
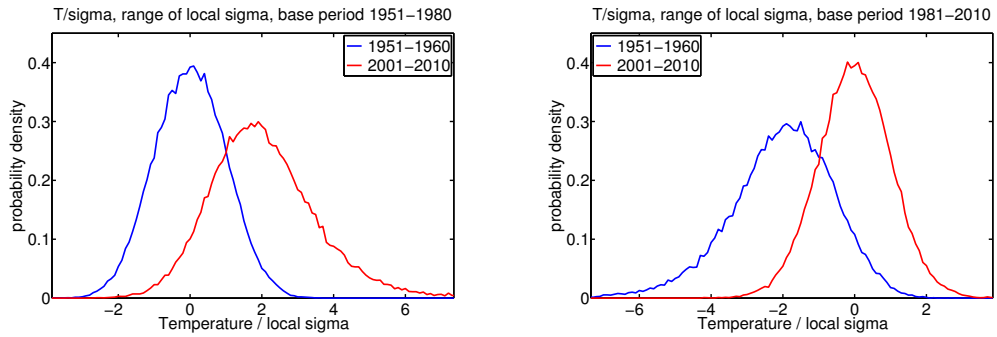


Figure 10: *From Hansen et al. [2012], figure 2. The sigma values for the period 1951-1980 for each location. The values have a strong location dependence and range from very small values of about 0.1 C to about 2 C with a spatial mean of 0.5 C.*



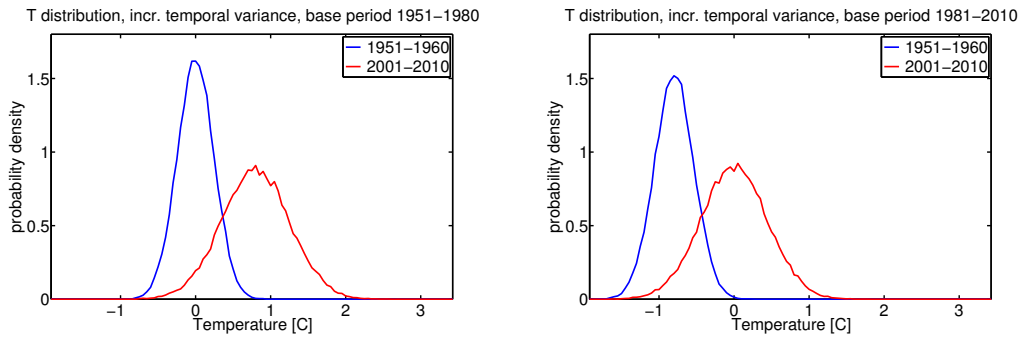
(a) *base period 1951-1980* (b) *base period 1981-2010*

Figure 11: *For a range of different sigma the T/sigma distributions outside of the base period are widened, as we expected due to the case study of two different sigma (Fig. 9), due to the 'pinch' effect that yields larger differences for T/sigma for different sigma only if T is large, which is most likely away from the base period, since by definition the mean of T is zero during the base period. The slight asymmetry of the distributions is due to the asymmetry of the sigma we are using, which is comparable to the actual asymmetric sigma distribution of real data.*

Increasing Temporal Variance

We turn now to a qualitatively different way to produce a widened T/sigma distribution as described earlier: increasing local temporal variability. This mechanism is independent of the 'pinch' effect created by choosing a base period. We simulate this effect by using two different seed sigma for simulating data, one small seed sigma (1951 - 1980) and one large seed sigma (1981 - 2010). These time-dependent seed sigmas are the same for all locations.

This mechanism also leads to a widened T distribution for a later decade (Fig 12a) if the mean anomaly of 1951-1980 is set to zero. This is because every local T distribution is widened in a later decade because the seed sigma for generating the data is larger then. However, in contrast to the other two effects, when choosing to set the local anomalies of 1981-2010 to zero we still generate a widened distribution for the later decade (Fig 12b). This will directly translate into a similar effect for T/sigma distributions, since dividing by the same sigma for each location will simply rescale the distributions.



(a) *base period 1951-1980* (b) *base period 1981-2010*

Figure 12: *An increase in every location's sigma over time will lead to a widened T distribution for later times, irrespective of the base period. This independence of the base period means this widening is not caused by the 'pinch' effect. The only difference between the distributions for different base periods is that the centered at different anomaly values, because the zero is defined at different times.*

Summary

We have seen that three different mechanisms can cause the T/sigma or T distributions to widen at later times. Some of these mechanisms, the ones based on the 'pinch' effect also show a strong width dependence on the choice of the base period.

Hansen et al. [2012] attribute the base period dependence of Fig. 2, for which they not only use a later base period but also the sigma from that later base period, to the greater variability at later times. The term 'greater variability at later times' can either mean that the a global average sigma were larger during 1981-2010 than in 1951-2010, which Hansen et al. demonstrated, or that the variabilities of the actual temperature distributions (not local anomalies relative to a common zero) are more different from each other at later times, which Hansen et al. did not demonstrate.

The first interpretation does not seem to be correct, because a larger average sigma by which anomaly values are scaled will only narrow each T/sigma distribution proportionally, as was shown by Hansen et a in their figure 4. Rather, the reason of the base period dependence has much more to do with the 'pinch' effect that forces each local anomaly to line up at the same mean over the base period, as we have discussed in this section.

Now that we understand the effects of the three mechanisms that can produce broadening of T/sigma distributions: a range of different sigma, a range of trends and an increasing temporal variability, we can apply these mechanisms to a seed global temperature anomaly function with a linear trend in 1981-2010, so we can more closely compare the simulated data to actual temperature data.

Global Temperature Trend - Linear Increase

For more realistic simulated data and to better compare to real data, we will now use a seed global anomaly function with a linear trend in 1981-2010 (Fig.4). We calculate the same quantities as previously for:

- a range of different trends (Fig.13)
- a range of different sigma (Fig.14)
- an increasing temporal variability sigma (Fig.15)

The range of trends causes gradual widening for both T/sigma (Fig.13a) and T distributions (Fig.13c) if the base period is 1951-1980. This mechanism also exhibits a base period dependence for both T/sigma (Fig.13b) and T distributions (Fig.13d) because for a later base period the distributions for times outside the base period are wider.

The range of sigma causes gradual widening for both T/sigma only (Fig.14a), not T distributions (Fig.14c) if the base period is 1951-1980. This widening also exhibits a base period dependence for T/sigma distributions (Fig.14b), i.e. the T/sigma distributions are always wider for times outside the base period.

For an increasing temporal variability both, the T/sigma and T distributions get wider at later times irrespective of the base period (Fig.15).

In the next section we will compare these signals to real data in order to determine which mechanism might be causing the broadening of the T/sigma distributions.

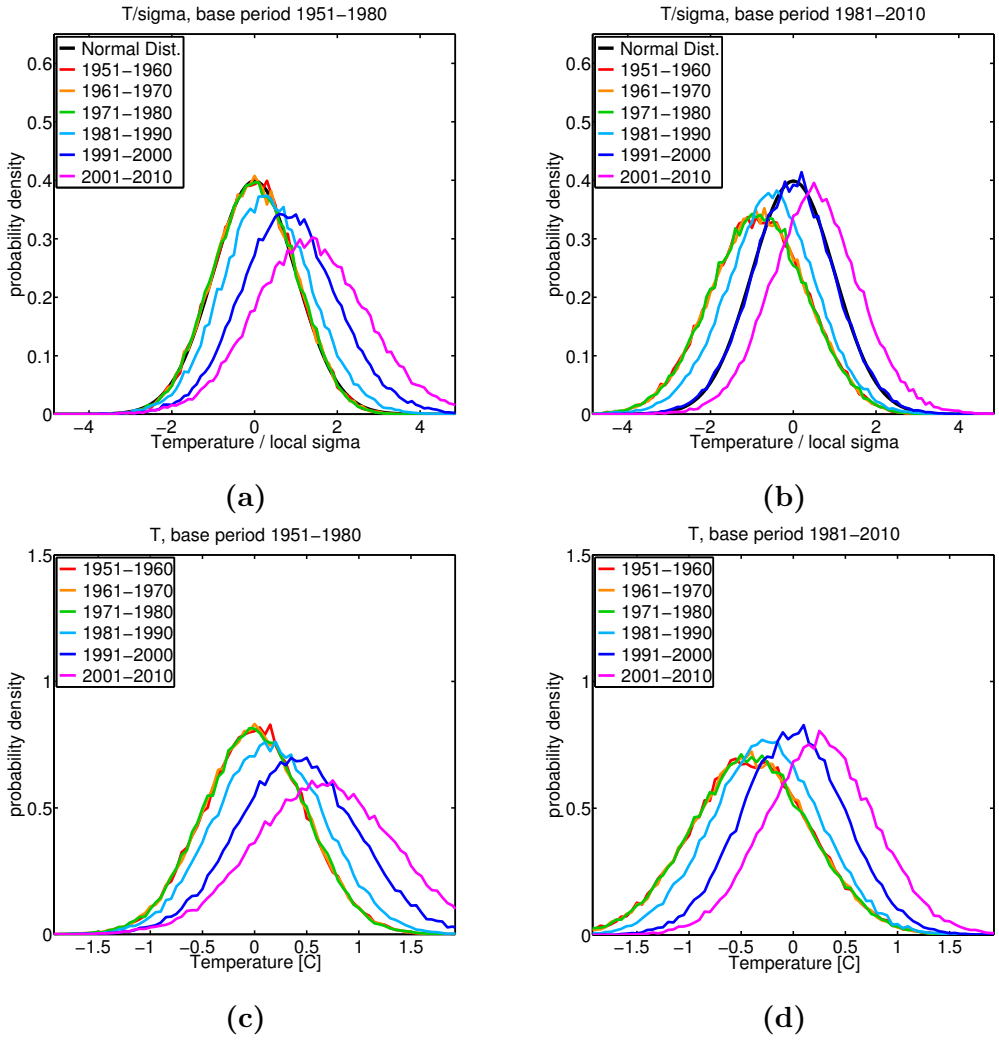


Figure 13: *A range of different trends can produce widening of the both T/sigma and T distributions and the widening shows base line dependence. (a) T/sigma distributions are wider at later times for the base period 1951-1980. (b) T/sigma distributions are wider at earlier times for the base period 1981-2010. (c) T distributions are wider at later times for the base period 1951-1980. (d) T distributions are wider at earlier times for the base period 1981-2010.*

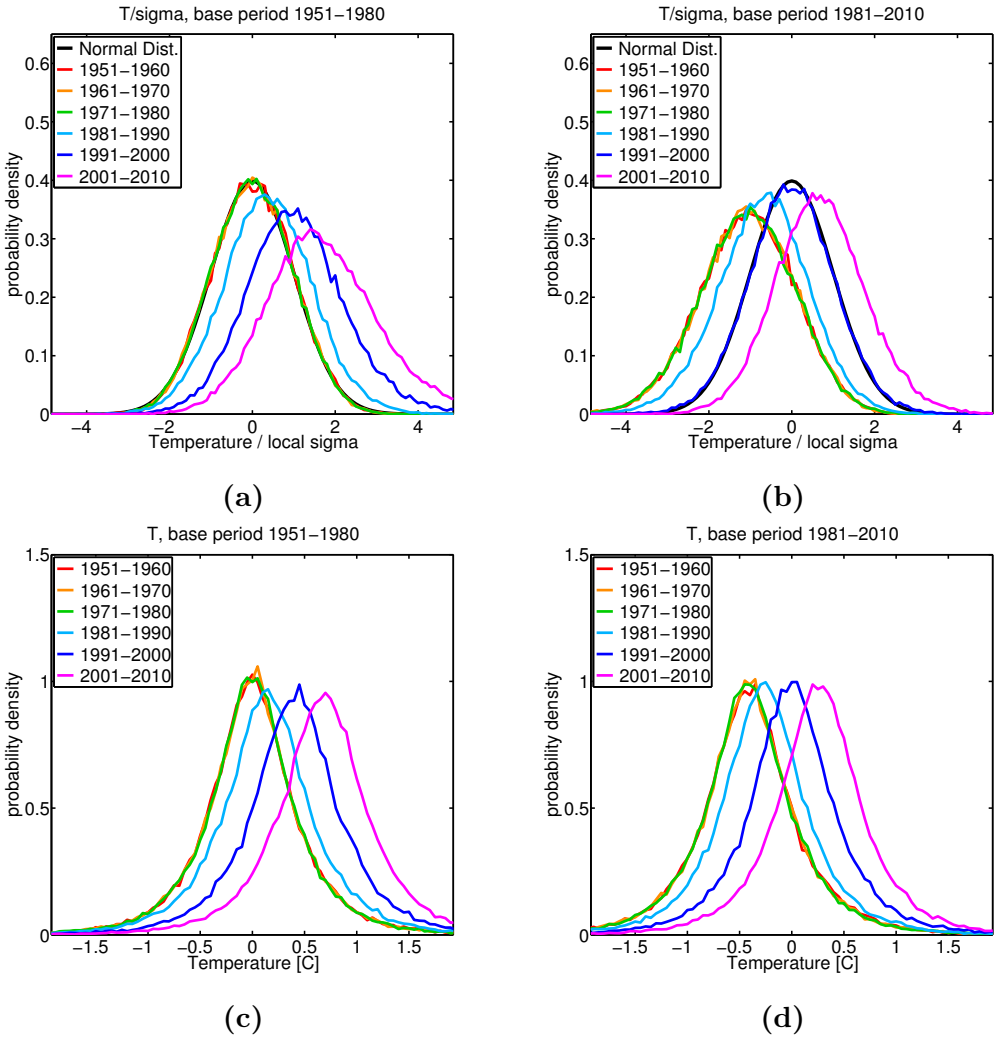
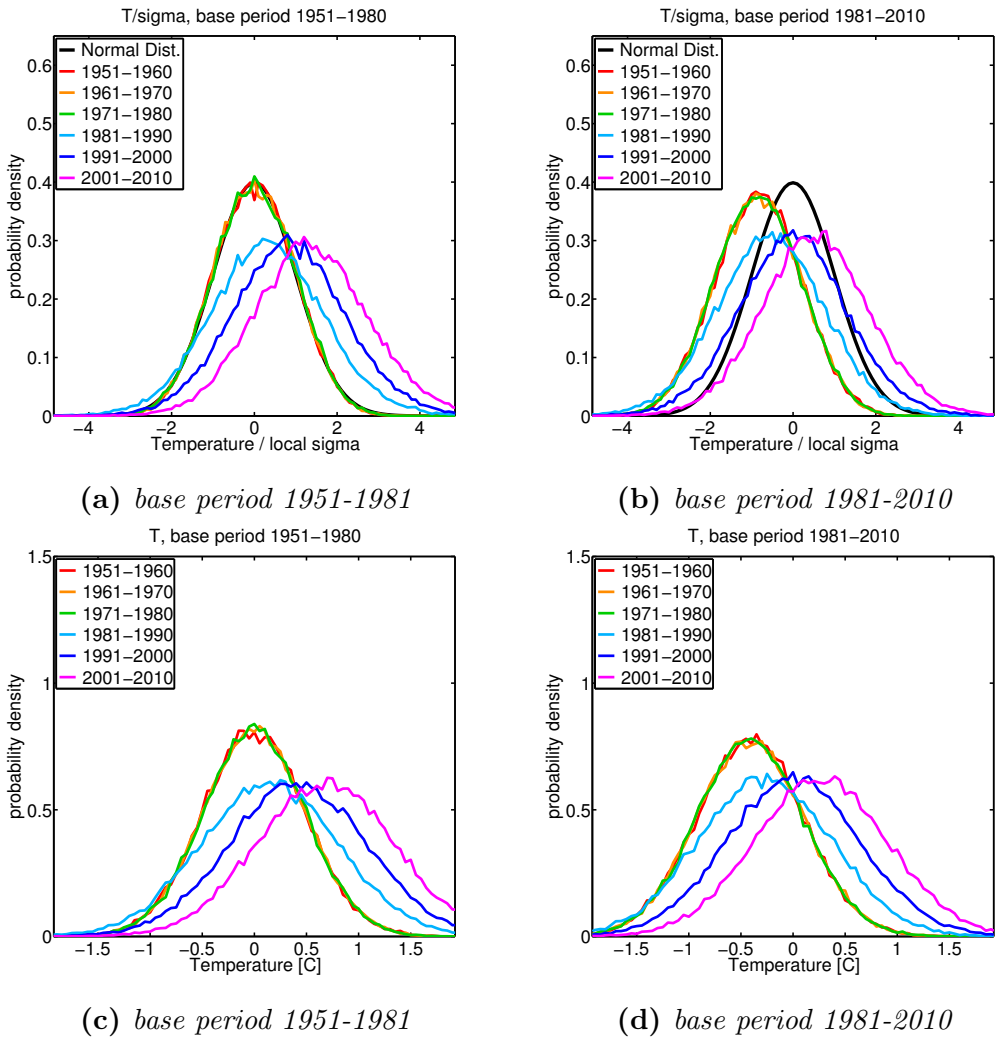


Figure 14: *A range of different sigma can produce widening of the T/sigma but not the T distribution and the widening shows base line dependence. (a) T/sigma distributions are wider at later times for the base period 1951-1980. (b) T/sigma distributions are wider at earlier times for the base period 19812010. (c) For base period 1951-1981, and (d) base period 1981-2010, T distributions don't change in width, but merely shift. It is interesting to note that the shape of the T/ distribution has longer tails than a normal distribution. This effect is also seen in the real temperature data (Fig.17), suggesting that it is due to the wide range of sigma. Each location can have normal distributed T anomalies for a 10 year period but with different widths sigma. The sum of these normal distributions will no longer resemble a normal distribution itself.*



(a) *base period 1951-1981* (b) *base period 1981-2010*

(c) *base period 1951-1981* (d) *base period 1981-2010*

Figure 15: *An increase in temporal variability causes each of these distributions to widen at later times, and there is no base period dependence.*

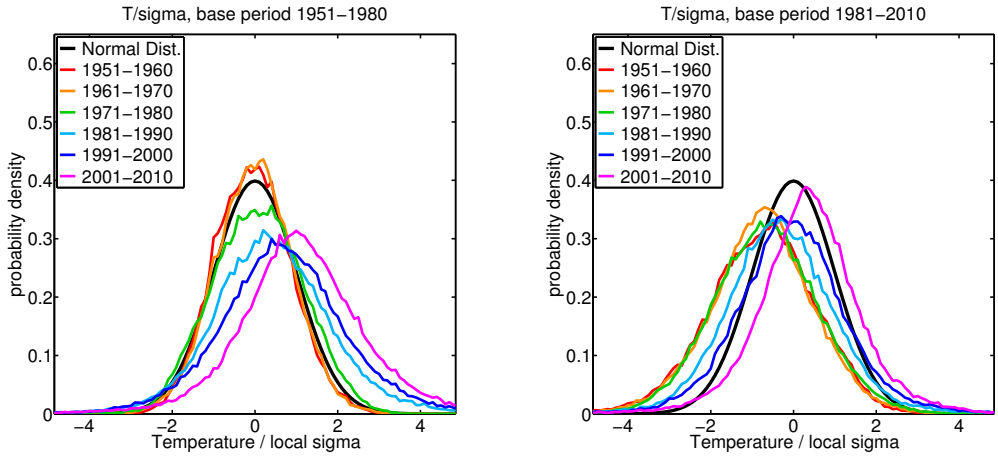
Temperature Data

We can now apply a similar analysis as above to the real data in order to find which signals are present. We use the same 250km GISS gridded data (Hansen et al. [2010]) used by Hansen et al. [2012]. As before we calculate the local standard deviations of 1951-1980 sigma, set the local mean of the base period, either 1951-1980 or 1981-2010, to zero.

We then create T/sigma distributions for each decade (Fig.16). We reproduce (Fig 16a) the main result of Hansen et al. [2012] (Fig 1): T/sigma distributions shift to higher values and get wider.

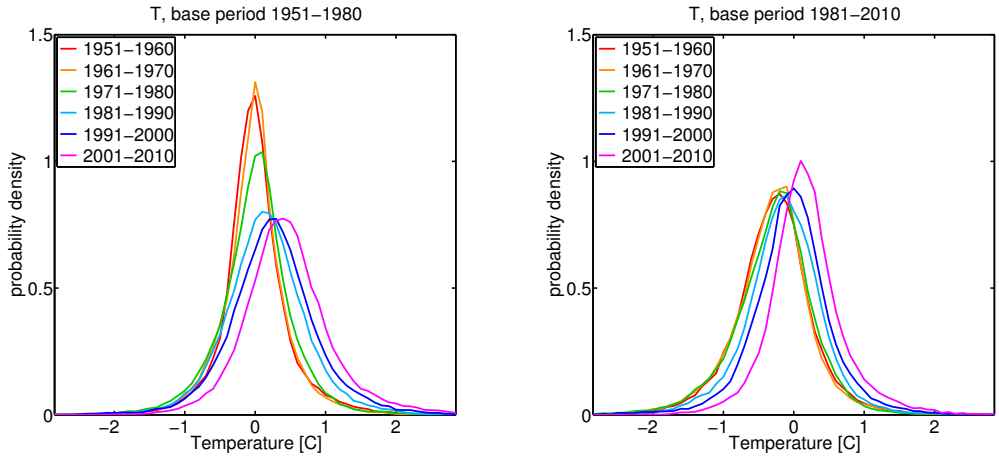
In addition, we find, that if we choose the base period to be 1981-2010, the distributions still shift but are not getting wider at later times (Fig 16b). Based on

the previous section we can now recognize this behavior of the distributions as an indication of the 'pinch' effect either due to a range of sigma or a range of trends, or both. In addition to T/sigma distributions, the T distributions for each decade (Figs 17) show a widening for later decades if the base period is 1951-1980 (Fig.17a), but no widening if the base period is 1981-2010 (Fig.17b). We can now recognize this behavior as a signal of the 'pinch' effect for a range of trends.



(a) *base period 1951-1980* (b) *base period 1981-2010*

Figure 16: *The T/sigma distributions for real temperature data. (a) For the base period 1951-1980, the distributions are getting wider in later decades. (b) For the base period 1981-2010 most distributions have the same width, but the later distribution is slightly narrower than the earlier ones. This is consistent with a widening mechanism based mainly on the 'pinch' effect.*



(a) *base period 1951-1980* (b) *base period 1981-2010*

Figure 17: *The T distributions for real temperature data. (a) For the base period 1951-1980, the distributions are getting wider in later decades. (b) For the base period 1981-2010 most distributions have the same width, but the latest distribution is slightly narrower than the earlier ones. This is consistent with a widening mechanism based mainly on the 'pinch' effect.*

We can conclude that the widening of the T/sigma distributions and the base period dependence of this widening seen by Hansen et al. [2012] is likely caused by a 'pinch' effect introduced by choosing local base periods. This effect then combines with a range of trends and most likely also a range of sigma to produce distributions that tend to be narrower during the base period and wider outside the base period. Any residual differences, i.e. for the later base period all distributions have similar widths rather than earlier ones being wider, may indicate a small influence of increased temporal variance, but it is clearly not the dominant factor.

In fact, in their figure 2 Hansen et al. [2012] show that temporal variability was slightly larger in 1981-2010 than in 1951-1980, and that half of this increase in temporal variability was in fact due to the temperature trend. However, firm conclusions about this are difficult, when the global data that is under-sampled, because, as Hansen et al. [2012] point out, this can lead to lower estimates for temporal variance, especially at early times, when temperature data was more sparse.

Conclusions

We set out to explain two surprising findings by Hansen et al. [2012]: For the base period 1951-1980 the T/sigma distribution widens in later times and the widening is dependent on the base period. We know from other studies, that there is a range of different local standard deviations and trends, and we have shown that these mechanisms can produce, both, the widening of the T/sigma distribution and the base period dependence of this widening found by Hansen et al. [2012]. Additionally, we have shown that a range of trends can explain the widening of temperature distributions that are not scaled by local standard deviations (Fig 17).

We made the argument that the 'pinch' effect, which is due to using a short base period to define climate, necessarily causes widening in T and T/sigma distributions. If one defines anomalies based on longer base periods, this effect can be avoided, as can be seen when Hansen et al. [2012] use the base period 1951-2010 (their figure 9), which shows no change in width of the distributions.

The method used by Hansen et al. [2012] yields information about the local changes in temperature, which have taken place since the local base period. This method will, in any reasonably noisy system, create wider global distributions at times outside the base period, irrespective of whether there is a global trend. Therefore, the number of 'hot' events, i.e. much higher local anomalies than during the base period, will always increase in such a system, especially if there also is a global average increase in T. The results of comparing the number of 3 sigma events at later times to the number of 3 sigma events during the base period, therefore, are difficult to interpret, because the 'pinch' effect will increase the number of such events at later times.

References

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